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A NEW HYBRID ANT BASED GENETIC ALGORITHM – FUZZY SHORTEST PATH PROBLEM

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ABSTRACT

Shortest Path (SP) problems in which, the determination of minimal path from source to the destination in the network graph G=$\{V,E\}$, have many dimensions in various fields of application. While considering a network graph, the arc length may represent distance, time, bandwidth or cost. But, in real life applications, there is certain uncertainty in the representation of real values as the arc length which in turn gives raise to fuzzy shortest path. In fuzzy shortest path problem, the edges are represented by fuzzy numbers and here we use generalized trapezoidal fuzzy numbers. The distance between the fuzzy edges is known to be fuzzy distance which comprises of centroid points, left spread and right spread. Genetic Algorithm (GA) is the most powerful among the optimization methods which involves 'natural selection' and the survival of the best individual to next generation. Ant Colony Optimization (ACO) works on the behaviour of the ants and the swarm intelligence. We propose a new hybrid ant based genetic algorithm with integrates the behaviour of ants and the genetic operators (specifically crossover and mutation operators). The implementation of proposed algorithm shows better convergence than the conventional algorithm.

KEYWORDS: Genetic Algorithm, Ant Colony, Generalized Trapezoidal Fuzzy Number, Shortest Path Problem

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1. INTRODUCTION & BACKGROUND

The shortest path problem has accustomed abundant absorption in the literature. Many applications such as communication, robotics, scheduling, transportation and routing, in which, shortest path is applied importantly. The realistic difficulty in representing the position of the vertices, so called edges leads to the introduction of fuzzy numbers [18], is frequently used to accord with uncertainties in a problem. We consider a directed network G = {V, E} where V represents the finite collection of vertices (nodes) and E represents the finite collection of directed edges. A source vertex and a destination vertex are specified and each edge length is represented by a generalized trapezoidal fuzzy number, and the length of a path is defined to be the fuzzy sum of edge lengths along the path. We are formulated so as in finding an optimized path from the source vertex to destination vertex while optimizing the fuzzy length of the path using the properties of generalized fuzzy numbers. Blue et al. [3] give taxonomy of network fuzziness that distinguishes five basic types combining fuzzy or crisp vertex sets with fuzzy or crisp edge sets and fuzzy weights and fuzzy connectivity.

The fuzzy shortest path problem was first analysed by Dubois and Prade [8]. They utilized the conventional shortest path algorithms, to treat the fuzzy shortest path problem. In order to make evolve the design of fuzzy systems, several metaheuristic learning algorithms are projected. One major improvement class uses evolutionary algorithms (EAs)

[17]. These algorithms are heuristic and random. They involve populations of individuals with a particular behavior like a biological development, like crossover or mutation. The most well-known biological process FSs are the genetic FSs [4],[10],[12],[14] that design FSs using Genetic Algorithms (GAs). Another class for FS style is that the Swarm Intelligence (SI) model [13], that could be a comparatively new optimization algorithm compared to EAs. The SI technique studies collective behavior in suburbanized systems. Its development was supported mimicking the social behavior of animals or insects in a shot to seek out the optima in the problem space. Another well-known SI is the ant-colony optimization (ACO) [5].

The ACO technique is impressed by real-ant-colony observations. It is a multiagent approach that was originally projected to resolve troublesome discrete combinatorial- optimization issues, like the traveling salesman problem (TSP) [6], [7]. In some studies, completely different ACO models were applied to FS design problems [9], [15]. In these studies, the antecedent-part parameters of an FS were manually appointed ahead. The consequent-part parameters were optimized in discrete space using ACO. ZainudinZukhriet. al. [19] proposed Genetic Ant Colony Optimization (GACO) which hybrids Genetic Algorithm (GA) and Ant Colony Optimization (ACO) uses the random selection of chromosome for 1st generation and pheromone. The population initialization has been made by both ants and GA and results better convergence than GA. Since solely the consequent-part parameters are optimized and also the optimization space is restricted to be discrete, the designed FSs are unsuitable for issues wherever high accuracy could be a major concern.

This paper is organized as follows. In section 2, some basic definitions are reviewed and explain the properties of generalized trapezoidal fuzzy numbers. Section 3 briefs the network terminology. Section 4 explains the proposed approach of Genetic Algorithm (GA). Section 5 deals with the implementation and results. And paper ends with the conclusion in section 6.

2. BASIC DEFINITIONS

The basic definitions of some of the required concepts are reviewed in this section.

2.1 Fuzzy Set

Let X be an universal set of real numbers R, then a fuzzy set is defined as

A= {[x,$\mu_{\tilde{A}}$(x)], x ϵ X}

This is characterized by a membership function: X → [0 1], Where, $\mu_A(x)$ denotes the degree of membership of the element x to the set A.

2.2 Characteristics of Generalized Fuzzy Number

A fuzzy set $\tilde{A}$ which is defined on the universal of discourse R, is known to be generalized fuzzy number if its membership function has the following characteristics

- $\mu_A : R \rightarrow [0,1]\ iscontinuous.$
- $\mu_A(x) = 0\ forallx \in (-\infty, a] \cup [d, \infty).$
- $\mu_A(x) isstrictlyincreasingon[a, b]andstrictlydecreasingon[c, d].$
- $\mu_A(x) = w, forallx \in [b, c], where\ 0 < w \leq 1.$

2.3 Membership Function of Generalized Trapezoidal Fuzzy Number

A generalized trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d;w) is known to be a generalized trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{w(x-d)}{(c-d)} & c \le x \le d \end{cases}$$

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then

a) $R(\tilde{A}) = \frac{w(a+b+c+d)}{4}$, b) $M(\tilde{A}) = \frac{w(b+c)}{2}$, c) divergence $D(\tilde{A}) = w(d-a)$, d) Left spread $LS(\tilde{A}) = w(b-a)$, e) Right spread $RS(\tilde{A}) = w(d-c)$

2.4 Fitness Function

The distance measure between the generalized trapezoidal fuzzy numbers $\tilde{A}$ (a_1,b_1,c1,d1;w_1) and $\tilde{B}$ (a_2,b_2,c_2,d_2;w_2) using centroid points (α, β) of $\tilde{A}$ is given by [10]

$$f_d(\tilde{A}, \tilde{B}) = \max\{|\alpha_{\tilde{A}} - \alpha_{\tilde{B}}|, |\beta_{\tilde{A}} - \beta_{\tilde{B}}|, |R(\tilde{A}) - R(\tilde{B})|, |LS(\tilde{A}) - LS(\tilde{B})|, |RS(\tilde{A}) - RS(\tilde{B})|\}$$

where $\alpha = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_1 a_2}{(a_4+a_3)-(a_1+a_2)}\right]$ and $\beta = \frac{1}{3}\left[\frac{a_3 - a_2}{(a_4+a_3)-(a_1+a_2)}\right]$

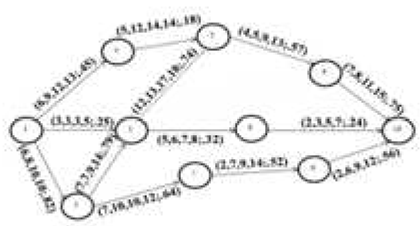
2.5 Addition of Fuzzy Numbers

Let $\tilde{A}$= (a_1,b_1,c_1,d_1;w_1) and $\tilde{B}$= (a_2,b_2,c_2,d_2;w_2) be two trapezoidal fuzzy numbers then the addition is defined by

$\tilde{A} \oplus \tilde{\boldsymbol{B}}$ = (a_1+a_2, b_1 +b_2, c_1+c_2, d_1+d_2; w_1+w_2)

3. NETWORK TERMINOLOGY

Consider the directed network G (V, E) consisting of a finite set of vertices V={1,2……n} and a set of m directed edges E $\subseteq V \times V$. Each edge is denoted by an ordered pair (i,j) where i, j$\in$ v and i$\neq$ j. In this network, we specify twovertices namely source vertex and the destination vertex. $\widetilde{d_{ij}}$ denotes the generalized trapezoidal fuzzy number associated with the edge (i,j). The fuzzy distance along the path P is given in section 2.6.



4. GENETIC ALGORITHM

Genetic Algorithm (GA) is a type of Evolutionary Algorithm (EA) which is based on the natural selection phenomenon. GA usually has an analogy to the randomness in solving a problem. It is comprised of generations where children are produced by the mating of the parents with genetic operators. Selection and reproduction to produce efficient generation is based on the random procedures, known to be natural selection.

4.1 Representation of an Individual (Chromosome)

Each chromosome is represented in binary representation and it is also important which represents the solution in the generations. The representation defines the path traversed and indirectly refers the fuzzy fitness of the chromosome. The number of bits used in representing chromosome is equal to the number of vertices in the network graph G= {V, E}. The vertex visited is represented by 1 and 0 represents that the vertex is not visited.

Here, we take 10 vertices network and the representation 1101100001 represents that the path traversed may be 1-2-4-5-10, 1-2-5-4-10, 1-4-2-5-10, 1-4-5-2-10, 1-5-4-2-10 and 1-5-2-4-10 depending on the existence.

4.2 Population Initialization

The initial population is generated randomly in usual GA and each chromosome represents the collection of edges which are represented by generalized trapezoidal fuzzy numbers explained in previous sections. The default population size 20 is used.

Initialization of the foremost generation is very important which implicates the convergence rate and existence of chromosomes in population. Thus we concentrate in upgrading the procedure of initializing population using Initializing Ant (IA) as an agent.

Genetic Algorithm (GA) possesses random selection of chromosomes in initializing the population, in which chromosomes may uncertain in the existence as solution space. Randomness may also produce discontinuous path as the chromosome in initializing population and leads to unhealthy and unfitted generations. Since initialization of population implicates the convergence rate, the chromosomes should exist, continuous and provide optimal fitness in solution space.

The identified problems explained above provides the importance of population initialization which is solved by using Initializing Ants (IA). The model is explained in our previous work [2].

4.3 Selection Operation

Selection operation is used in initialization process and parent selection for crossover operation. Various selection operations involve Roulette wheel selection, Random selection, Rank selection, Tournament selection and Boltzmann selection [16].

Here we choose distance measure considering rank, divergence, left spread and right spread of generalized trapezoidal fuzzy numbers explained in previous section as the fitness function. As the fuzzy shortest path problem is considered, minimum will survive by the selection the chromosome values.

The distance measure between every two edges is calculated and obtained distance is summed up throughout the path. The chromosome having minimum distance (fitness) will have more probability to be selected in the generation.

4.4 Crossover& Mutation Operation

Crossover operator mates two parent chromosomes and produces children which comprise the essence of two parent chromosome mated. Crossover operation is mainly categorised into two single point and multi point crossover. The single point crossover has single crossover site whereas multi point crossover has more than single crossover site. There are also some advanced multipoint crossover methods [16] and here we use a type of two point crossover technique.

The conventional mutation operator performs the minute changes of the reproduced child randomly under a certain rate which undo the degradation of the population due to crossover operationwith crossover rate of 0.5.

There were many mutation operations for binary and real integers. Here we choose binary mutation that may be bit flipping, insertion, interchanging, reciprocal exchange, inversion and others [1].

The proposed method combines both the crossover and mutation in single operation. Since only the chromosomes having continuous path exists in the generation, we should also maintain its existence in crossover and mutation operation.

The parents having continuous path is connected through a mutator without affecting the existence (continuous) of the chromosomes. Bit flipping or addition is used as mutator in connecting the both parents to form next generations.

Consider an example with two parent chromosome A (1011001111) and B (1001101001). A two point crossover has to be carried out with a rate of 0.5 and mutation at 0.1.

Parent 1 1 – 2 – 7 – 8 – 10 Parent 2 1 – 4 – 5 – 9 – 10

Child 1 – 2 – 4 – 5 – 9 – 10
(Crossover)

Child 1 – 2 – 3 – 4 – 5 – 9 – 10
(Mutation)

Here, parents are merged with the connector (addition mutator) node 3 for obtaining continuous path. Bit flipping is also used to obtain continuous path.

4.5 Termination Condition

Termination condition produces the optimal solution through the convergence. Mostly termination condition will be the maximum number of generations. Other conditions are the idealness of the chromosomes in the generation. In order to test the algorithm, maximum number of generations can be used as termination condition which clearly represents the convergence of the algorithm.

Here, idealness of the chromosomes is considered as termination condition because of the usage trapezoidal fuzzy numbers and uncertainty in real numbers. When no change in the optimal fitness (minimal) and the idealness of the chromosomes in generations for at least 5 generations, then the algorithm reaches the termination condition. The network in section 3 after applying the proposed algorithm results the shortest path 1 – 3 – 6 – 10.

Algorithm

Step 1: Generate the network with vertices and edges with generalized trapezoidal fuzzy numbers.

Step 2: Population is initialized using Initializing Ants (IA) as proposed in our previous work [17].

Step 3: Apply cross over process with the parent chromosomes selected using distance measure with the rate of 0.5 as explained in section 4.4.

Step 4: Mutation is carried out with a rate 0. 1 randomly as explained in section 4.4.

Step 5: when fitness of child is better than parent, child moves for next generation.

Step 6: Repeat the steps 3 to 5 till reaches the termination condition as explained in section 4.5.

Step 7: Report the chromosome with minimal fitness as the solution after termination condition.

5. NUMERICAL EXAMPLE

We Consider the network G= {V, E} of vertices (n=10) represented in section 3. According to the assumption, we take 2^5 ($2^{10/2}$) IA to serve for the finding of shortest path as initializing population of Genetic Algorithm (GA). Every edge is represented by the generalized trapezoidal fuzzy number. The fitness, ranking and properties of generalized trapezoidal fuzzy number in which described in previous chapters, are used to calculate.

The population initialization is elaborately explained in our previous work [2]. The distance measure will act as a fitness value of the chromosome $f_d(\tilde{A}, \tilde{B})$ given in section 2.4. Let consider two generalized trapezoidal fuzzy edges A(3,3,3,5;.25) and B(5,6,7,8;.32).

$$\alpha_A = 3.6667, \beta_A = 0.3334, R(A) = .875, LS(A) = 0, RS(A) = .5$$

$$\alpha_B = 6.5, \beta_B = .4167, R(B) = 2.08, LS(B) = .32, RS(B) = .32$$

$$f_d(\tilde{A}, \tilde{B}) = max|2.8333, .0833, 1.205, .32, .18\,| = 2.8333$$

Table 5.1: Calculation $f_d()$ of Path 1-3-6-10 (Continuous Path)

Path	Next Vertex	$f_d(\tilde{A}, \tilde{B})$(Section 2.6)
1	3	0
1-3	6	0+2.833=2.833
1-3-6	10	2.833+2.214=5.047

Table 5.2: Calculation $f_d()$ of Path 1-4-5-9-10 (Continuous Path)

Path	Next Vertex	$f_d(\tilde{A}, \tilde{B})$(Section 2.6)
1	4	0
1-4	5	0+2.475=2.475
1-4-5	9	2.475+3.033=5.508
1-4-5-9	10	5.508+4.091=9.599

From the table 5.1 and 5.2, the distance measure (fitness) of two continuous paths A1-3-6-10 and B 1-4-5-9-10 is shown and our selection is based on the comparison of distance measure. Thus while comparing these paths, it shows to be A<B and minimal fitness A (5.047) is selected for the next generations.

6. IMPLEMENTATION & RESULTS

The implementation is carried out in Matlab 8.1 (R_{2013a}) 32 bit student version. The implementation is extended with our previous work and selection of valid path is controlled using adjacency matrix.

The network G={V,E} of 30 nodes with the edges of generalized trapezoidal fuzzy number is initialized. The algorithm is implemented as per the given description and demonstrated numerical calculation.

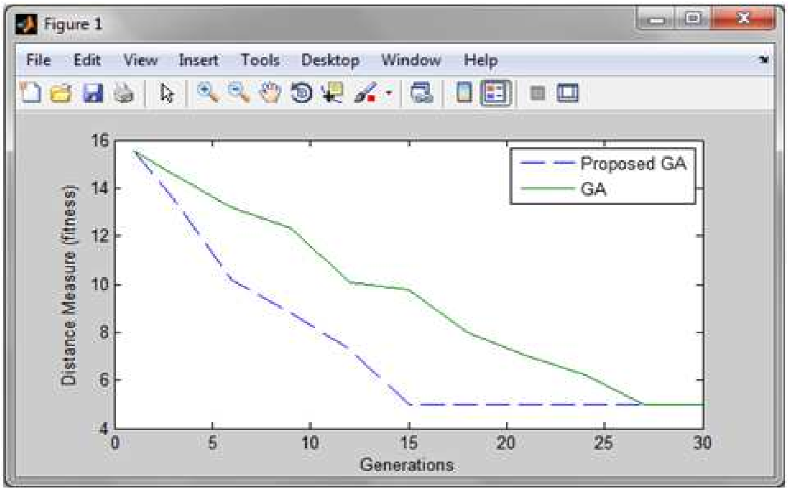


Figure 5.1: Comparison on Fitness Measures with Generations

From the figure 5.1**,** it is clear that the distance measure methoddepends on the rank,mode, divergence, left spread and right spread. The path at which all the components attain equilibrium is considered to be the shortest path. Here, generations around 15 – 27in which chromosomes possess constant fitness andidealness in the distance measure generations and the path obtained is considered to be shortest path.

The huge difference between the algorithms is because of the selection of continuous path for the generations. The proposed crossover and mutation always produces the valid chromosomes with continuous path whereas conventional GA may fails with its crossover and mutation operation in initialization and further generations.

7. CONCLUSIONS

The Shortest Path (SP) problem in many applications is uncertain in parameters (Distance, Range, etc.). Hence, there occurs the necessity of fuzzy numbers for uncertain parameters. We propose a new hybrid ant based optimization algorithm along with the generalized trapezoidal fuzzy numbers and distance measure method. The result clears that the proposed new hybridGenetic Algorithm (GA) comprises the shortest path with optimalgenerations using swarm intelligence and all the chromosomes in every generations is continuous and existed in the network.

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